11.10

FIND THE FIRST ~~3~~ 4 NON-ZERO TERMS IN THE TAYLOR SERIES FOR $f(x) = \frac{\cos x}{1+x}$

$$= \cos x \cdot \frac{1}{1+x}$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots$$

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 + \cdots$$

$$\frac{1}{1-(-x)}$$

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 + x^4$$

$$\cos x = 1 + 0x - \frac{x^2}{2!} + 0x^3 + \frac{x^4}{4!}$$

$$\frac{1}{1+x} \cdot \cos x = 1 - x + \left(-\frac{1}{2!} + 1\right)x^2 + \left(\frac{1}{2!} - 1\right)x^3$$

$$= 1 - x + \frac{1}{2}x^2 - \frac{1}{2}x^3 + \cdots$$

$$(a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots)$$

$$* (b_0 + b_1 x + b_2 x^2 + b_3 x^3)$$

$$= a_0 b_0 + (a_0 b_1 + a_1 b_0)x + (a_0 b_2 + a_1 b_1 + a_2 b_0)x^2 + (a_0 b_3 + a_1 b_2 + a_2 b_1 + a_3 b_0)x^3$$

OR $\dfrac{\cos x}{1+x} = \dfrac{1 - \frac{x^2}{2!} + \frac{x^4}{4!} \cdots}{1+x}$

$$
\begin{array}{r|l}
 & 1 - x + \tfrac{1}{2}x^2 - \tfrac{1}{2}x^3 + \tfrac{13}{24}x^4 \\
\hline
1+x & 1 \qquad - \tfrac{1}{2}x^2 \qquad + \tfrac{1}{24}x^4 \\
 & \underline{-1 - x} \\
 & -x - \tfrac{1}{2}x^2 \\
 & \underline{+x + x^2} \\
 & \tfrac{1}{2}x^2 \\
 & \underline{-\tfrac{1}{2}x^2 - \tfrac{1}{2}x^3} \\
 & -\tfrac{1}{2}x^3 + \tfrac{1}{24}x^4 \\
 & \underline{+\tfrac{1}{2}x^3 + \tfrac{1}{2}x^4} \\
 & \tfrac{13}{24}x^4
\end{array}
$$

FIND THE FIRST ~~4~~ 3 TERMS OF THE MACLAURIN SERIES FOR $f(x) = \tan x = \dfrac{\sin x}{\cos x}$

$$
\begin{array}{r|llll}
 & x & +\frac{1}{3}x^3 & +\frac{2}{15}x^5 & \\
\hline
1 - \frac{x^2}{2!} + \frac{x^4}{4!} & x & -\frac{x^3}{3!} & +\frac{x^5}{5!} & -\frac{x^7}{7!} \\
 & -x & +\frac{x^3}{2!} & -\frac{x^5}{4!} & \\
\hline
 & & \frac{1}{3}x^3 & -\frac{1}{30}x^5 & -\frac{x^7}{7!} \\
 & & -\frac{1}{3}x^3 & +\frac{1}{6}x^5 & -\frac{1}{72}x^7 \\
\hline
 & & & \frac{2}{15}x^5 &
\end{array}
$$

$$-\frac{1}{6} + \frac{1}{2} = \frac{1}{3}$$

$$\frac{1}{5 \cdot 4!} - \frac{1}{4!} = \frac{1-5}{5 \cdot 4!} = \frac{-4}{5 \cdot 4!}$$

$$-\frac{1}{30} + \frac{1}{6} = \frac{-1+5}{30} = \frac{4}{30} = \frac{2}{15}$$

$$\tan x \approx x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + \dots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}$$

$$\cos \frac{\pi}{2} = 1 - \frac{(\frac{\pi}{2})^2}{2!} + \frac{(\frac{\pi}{2})^4}{4!} - \frac{(\frac{\pi}{2})^6}{6!} + \cdots$$

FIND $1 - \frac{\pi^2}{4 \cdot 2!} + \frac{\pi^4}{16 \cdot 4!} - \frac{\pi^6}{64 \cdot 6!} + \cdots = \cos \frac{\pi}{2} = 0$

$$1 - \frac{(\frac{\pi}{2})^2}{2!} + \frac{(\frac{\pi}{2})^4}{4!} - \frac{(\frac{\pi}{2})^6}{6!}$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} \quad \text{IF} \quad x = \frac{\pi}{2}$$

11.10/11.11.

REMAINDERS FROM TAYLOR SERIES

$$f(x) \approx T(x) = \sum_{n=0}^{\infty} c_n (x-a)^n$$

$$T_n(x) = \sum_{i=0}^{n} c_i (x-a)^i$$

TAYLOR POLYNOMIAL
OF DEGREE n

IF WE USE $T_n(x)$ TO APPROXIMATE $f(x)$
HOW FAR COULD $T_n(x)$ BE FROM $f(x)$?

$$R_n(x) = f(x) - T_n(x) \longrightarrow f(x) = T_n(x) + R_n(x)$$

REMAINDER/ERROR FROM USING T_n TO APPROXIMATE f

$$T(x) = \sum_{i=0}^{\infty} c_i (x-a)^i$$

$$T_n(x) = \sum_{i=0}^{n} c_i (x-a)^i$$

$$R_n(x) = T(x) - T_n(x) = \sum_{i=n+1}^{\infty} c_i (x-a)^i$$

SINCE WE USE $T_n(x)$ TO APPROXIMATE $f(x)$ WHEN WE DON'T KNOW $f(x)$

THEREFORE WE DON'T KNOW $R_n(x)$